

The Treewidth of Smart Contracts*

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ABSTRACT

Smart contracts are programs that are stored and executed on the Blockchain and can receive, manage and transfer money (cryptocurrency units). Two important problems regarding smart contracts are formal analysis and compiler optimization. Formal analysis is extremely important, because smart contracts hold funds worth billions of dollars and their code is immutable after deployment. Hence, an undetected bug can cause significant financial losses. Compiler optimization is also crucial, because every action of a smart contract has to be executed by every node in the Blockchain network. Therefore, optimizations in compiling smart contracts can lead to significant savings in computation, time and energy.

Two classical approaches in program analysis and compiler optimization are *intraprocedural* and *interprocedural* analysis. In intraprocedural analysis, each function is analyzed separately, while interprocedural analysis considers the entire program. In both cases, the analyses are usually reduced to graph problems over the control flow graph (CFG) of the program. These graph problems are often computationally expensive. Hence, there has been ample research on exploiting structural properties of CFGs for efficient algorithms. One such well-studied property is the treewidth, which is a measure of tree-likeness of graphs. It is known that intraprocedural CFGs of structured programs have treewidth at most 6, whereas the interprocedural treewidth cannot be bounded. This result has been used as a basis for many efficient intraprocedural analyses.

In this paper, we explore the idea of exploiting the treewidth of smart contracts for formal analysis and compiler optimization. First, similar to classical programs, we show that the intraprocedural treewidth of structured Solidity and Vyper smart contracts is at most 9. Second, for global analysis, we prove that the interprocedural treewidth of structured smart contracts is bounded by 10 and, in sharp contrast with classical programs, treewidth-based algorithms can be easily applied for interprocedural analysis. Finally, we supplement our theoretical results with experiments using a tool we implemented for computing treewidth of smart contracts and show that the treewidth is much lower in practice. We use 36,764 real-world Ethereum smart contracts as benchmarks and find that they have an average treewidth of at most 3.35 for the intraprocedural case and 3.65 for the interprocedural case.

*A longer version of this paper is available at [11] (<https://repository.ist.ac.at/1070/>).

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CCS CONCEPTS

• **Applied computing** → **Secure online transactions**; • **Software and its engineering** → **Formal software verification**;

KEYWORDS

Smart Contracts, Parameterized Algorithms, Treewidth, Blockchain, Compiler Optimization, Program Analysis, Control Flow Graphs

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1 INTRODUCTION

Smart Contracts. Bitcoin scripts [5] initiated the idea of encoding semantics in cryptocurrency transactions. These scripts were used to implement simple contracts, such as escrows [49]. However, one can encode more complicated programs in a cryptocurrency transaction, in order to implement complex financial agreements, such as credit reporting [29] or decentralized autonomous organizations [43]. Such programs are called smart contracts. A smart contract has its own dedicated memory and can be programmed to receive, manage and transfer cryptocurrency units [37]. Users (and other contracts) can interact with the contract by calling one of its functions. Each such function call is handled in the same way as a transaction. The Blockchain protocol provides a global consensus about the state of each contract. This includes consensus about the code of the contract, the state of its memory, and the results of interactions with the contract. Note that after a contract's code is stored on the Blockchain, it is immutable, and the only way to interact with it is to call its functions, which behave as programmed.

Value of Contracts. Smart Contracts hold and manage a considerable amount of funds. At the time of writing, on Ethereum Blockchain alone, there are over a million instances of deployed smart contracts, holding billions of dollars of funds [44].

Importance of Formal Analysis. Given the unmalleability of data stored on the Blockchain, a contract's code cannot be amended after its deployment. Similarly, all transactions stored in a Blockchain are irreversible. On the other hand, contracts handle a huge amount of funds. Therefore, to avoid significant financial losses, bugs in smart contracts must be detected before deployment [4, 13, 16, 38]. For example, in one catastrophic case, called the DAO attack [43], an attacker stole more than 50 million dollars from a contract. There are also a variety of proposed best-practices and design patterns for minimizing the damages when a contract fails (e.g. see [19]).

Importance of Optimization. To achieve consensus, each time a function is called, all nodes of the network have to verify the results

by running the function [47]. This means that each function call is executed and verified by tens of thousands of nodes in parallel [27]. Therefore, any optimization in the compilation process can lead to a massive overall saving of time and energy.

Importance of Quantitative Analysis. In many scenarios, qualitative analysis is not enough for smart contracts [16]. For example, absolute safety against any attacks might be impossible or very costly. In such scenarios, one would like to find a bound on the potential or expected economic consequences of an attack. Moreover, quantitative analysis approaches, such as [18], can be used to find the expected execution costs of a contract.

Control Flow Graphs. Many problems in program analysis, model checking and compiler optimization can be reformulated as graph problems [36]. In such cases, the underlying graph is usually the Control Flow Graph (CFG) of the program that is being analyzed or optimized [34, 36]. See Section 2.1 for a formal definition of CFGs. In such analyses, one can either consider the problem over individual functions and hence create a separate CFG for each function, or attempt to solve the problem on a global control flow graph (GCFG) of the entire program. The former approach is called *intraprocedural* analysis, and the latter is *interprocedural* analysis.

Exploiting Structural Properties. The graph problems arising from formal program analysis and compiler optimization are often computationally expensive and even NP-hard in many cases. Hence, there has been ample research on exploiting the structural properties of the underlying CFGs to obtain faster algorithms [3]. An extensively-studied parameter that has been applied successfully to these problems is the treewidth [8, 14, 17, 31, 35, 39, 45]. See section 2.3 for some motivating examples.

Treewidth. Treewidth [40] is a well-studied graph parameter that provides a measure of tree-likeness of graphs [7]. Trees and forests are the only graphs with a treewidth of 1 and, informally, a lower treewidth means that the graph has more resemblance to trees. The significance of treewidth in the design of algorithms stems from the fact that many NP-hard graph problems are fixed-parameter tractable when parameterized by the treewidth, i.e. can be solved efficiently on graphs that have small treewidth [6, 7, 12, 22, 23, 28, 30]. Specifically, one can apply a bottom-up dynamic programming technique on such graphs in a manner very similar to trees [6]. For the formal definition of treewidth, see Section 2.4.

Treewidth of Control Flow Graphs. In [45], it was established that the intraprocedural CFGs of goto-free structured programs in several languages, including C and Pascal, have a treewidth of at most 6. This result provided a basis for the fast formal analysis and compiler optimization algorithms mentioned earlier.

Our Contribution. We study the treewidth of goto-free Ethereum smart contracts. We focus on two contract programming languages, namely Solidity and Vyper. We obtain the following results:

- (i) **Theoretical Results.** First, for intraprocedural analysis, we show that CFGs of smart contracts have a treewidth of at most 9 (Theorem 3.1). This is similar to the result for classical programs. Second, in contrast to classical programs, for global (interprocedural) analysis, we show that the global CFGs of Vyper smart contracts are finite, have a treewidth of at most 10 (Theorem 4.1), and their tree decompositions can be succinctly

represented (Theorem 4.2). The same result also holds for non-recursive Solidity smart contracts. Hence, unlike classical programs, for smart contracts, solving global variants of many formal analysis and compiler optimization tasks is no harder than the local (intraprocedural) variants.

- (ii) **Experimental Results.** On the experimental side, we implemented a tool for obtaining tree decompositions of CFGs and GCFGs of Solidity smart contracts. We analyzed 36,764 real-world Solidity smart contracts currently deployed on the Ethereum Blockchain. The results showed that (i) no real-world Solidity smart contract used recursion, hence all of our theoretical results for Vyper contracts carry on to real-world Solidity contracts, (ii) in case of CFGs, the average treewidth was 3.35 and the bound 9 was never met, (iii) in case of GCFGs, all analyzed contracts were shown to have finite and succinctly-representable GCFGs. The average GCFG treewidth was 3.65 and the bound 10 was never met in practice.

Significance of Our Results. There are two important takeaways from our results. First, much like classical programs, the bounded treewidth property of the CFGs of smart contracts can be exploited for intraprocedural compiler optimization and program analysis tasks. Second, unlike classical programs, smart contracts have succinctly representable GCFGs with small treewidth. This means that treewidth can be exploited for the same optimization and formal analysis problems in an *interprocedural* setting, rather than just intraprocedural analysis. This can potentially lead to much more powerful algorithms and tools for the analysis of smart contracts. Note that, while exploiting treewidth in general programs is well-studied and supported by tools such as [15], current approaches for analyzing smart contracts, such as [16, 38, 46], do not exploit the treewidth or any other structural property.

2 PRELIMINARIES AND MODELING

In this section, we provide some basic definitions, define an abstract programming language that we are going to use in the rest of the paper, and review previously-known results.

2.1 Programs and Graphs

Control Flow Graphs (CFGs). The Control Flow Graph (CFG) of a program is a directed graph whose paths model the execution traces of the program [1]. There are many slightly different variations of CFGs. The nodes of a CFG can correspond to statements in the program, or basic blocks, or other subdivisions of the code. We will follow the node structure used in [45] (explained below).

Abstract Programming Language. We define an extension of the STRUCTURED programming language [45] to capture the general properties of goto-free programs, abstract away the details that are not relevant to the treewidth of the CFG, and obtain general results that will then be instantiated for specific programming languages. We call this extension ES (Extended STRUCTURED). The differences between ES and STRUCTURED are that (i) to enable interprocedural analysis, ES does not abstract away function calls, and (ii) ES allows several exit types for every function. Informally, each function in ES is like a program in STRUCTURED.

ES. A program in ES is a set of functions. Each function starts with the keyword `function`, followed by its name, followed by a sequence of statements in the function, and finally ends with the keyword `endfunction`. The statements are of these types:

- Conditional statements, `if-then-endif` and `if-then-else-endif`,
- A general loop structure `loop-endloop` that does not normally end on its own,
- A break statement, ending the innermost surrounding loop,
- A continue statement that goes to the next iteration of the innermost surrounding loop,
- A return statement that terminates the current function and returns control to the parent function,
- Several *types* of exit statements, $exit_1, exit_2, \dots, exit_k$, each of which terminates the program,
- Function call statements call `<function-name>`, and
- Atomic statements shown by the keyword `atomic`. By an `atomic`, we mean a statement that does not affect the flow of the program and can be abstracted away, e.g. an assignment.

All statements that lack an “end” in their structure must be followed by a semicolon. Intuitively, the different types of exit statements correspond to the different types of termination that can happen in a smart contract (see Section 2.2). We assume that a `break` or `continue` that is not surrounded by a loop acts as a return. As in `STRUCTURED`, the conditions of `if` statements can be boolean expressions consisting of atomic boolean variables a, b, \dots , and the operators `and` and `or`. We also assume short-circuit evaluation.

Nodes of a CFG. We construct the CFG for each function separately. In the CFG, we put one vertex for each exit type and one for every word of the ES code, except for function names¹. Note that this includes semicolons. There is an edge between two vertices, if their corresponding words are in the same function and can be visited consecutively in some execution of the ES program, i.e. in CFGs we treat call statements in the same manner as atomic statements. Figure 1 shows an example ES program together with its CFG. The nodes corresponding to exit types are shown by hexagons. We will refer to them as hexagonal nodes in the sequel.

Call Graphs and Recursive Functions. The call graph [41, 42] of a program is a directed graph, in which there is one vertex corresponding to each function, and there is a directed edge from a vertex u to a vertex v , if the function corresponding to u , at some point, calls the function corresponding to v . For example, the call graph of the program in Figure 1 has a single edge $f1 \rightarrow f2$. If the call graph contains a cycle, we say that the program is *recursive* [25]. Otherwise, the program is said to be *nonrecursive* or *simple*.

CFGs are usually sufficient for intraprocedural analysis. However, in order to perform interprocedural analysis the data encoded by the call graph becomes necessary, too. This complicates the situation, given that the analysis problems are now reduced to graph problems over two graphs with nontrivial interactions. We can mitigate this problem by using a Global (interprocedural) CFG. Intuitively, the process for obtaining the GCFG is similar to repeated inlining [10].

Global Control-Flow Graphs (GCFGs). The GCFG of a program is obtained from its CFG by repeatedly expanding the function call

¹This definition of nodes is very fine-grained. However, Lemma 2.2 shows that one can contract any two vertices to get a coarser CFG, without increasing the treewidth.

```
function f1
  if a or b then
    atomic;
    return;
  else
    call f2;
  endif
endfunction

function f2
  loop
    if c then
      break;
    endif
    atomic;
    if d then
      continue;
    endif
    if e then
      exit_1;
    endif
  endloop
endfunction
```

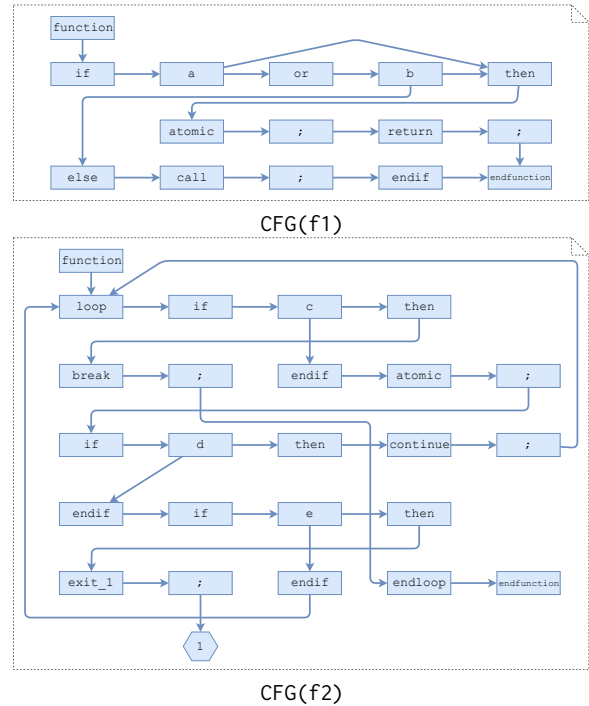


Figure 1: An ES program (top) consisting of two functions $f1, f2$ and the CFG of $f1$ (middle) and that of $f2$ (bottom). Exit nodes are shown by hexagons.

nodes with copies of the CFG of the function that is being called at that point. This process is continued as long as there are unexpanded function call nodes remaining in the graph. Concretely, in case of ES programs, a function call `call f;` is expanded by putting a copy of the CFG of f between the call node and its corresponding semicolon. Specifically, the edge between the call node and its semicolon node is removed, the call node is connected to the function node of the copy and the `endfunction` node is connected to the semicolon. This is illustrated in Figure 2. Intuitively, each node of the GCFG encodes not only a point of the code, but also the functions that are on the stack when reaching that point. So, the GCFG of a program is finite iff the program is simple.

2.2 Ethereum

Ethereum [9] is a cryptocurrency platform that allows smart contracts of arbitrary, i.e. Turing-complete, complexity. Smart contracts

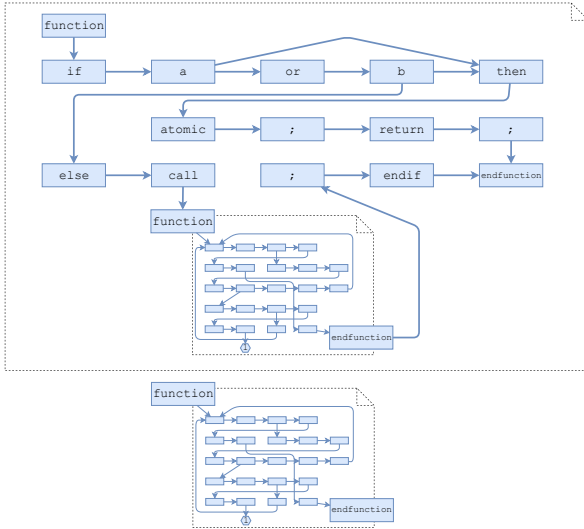


Figure 2: The GCFCG of the program in Figure 1.

are fundamental to Ethereum, to the extent that Ethereum creators often call it a distributed computing platform [9], considering smart contracts as “decentralized applications” and the currency, Ether, as a mechanism of payment in exchange for consensus and computation. Therefore, it is of paramount importance that every node in the Ethereum network has the exact same understanding about the meaning (semantics) of any piece of smart contract code. This is achieved by means of a virtual machine.

Ethereum Virtual Machine (EVM). The EVM is the runtime environment for Ethereum programs (smart contracts). It is a Turing-complete stack machine programmable with a formally defined Bytecode format [47]. Every node in the Ethereum network runs an instance of the EVM. This ensures that all nodes are in consensus about the results of any transaction (function call). However, the downside is that every function call has to be executed by every node. Therefore, there are considerable costs associated with computations in smart contracts and the network might be attacked by spammers who intend to drain its computational power. Ethereum addresses this problem using the concept of Gas.

Gas. Every operation in the EVM has an associated cost, in Ether, which is roughly correlated with the amount of computational power it uses [47]. This cost is called gas. When a user calls a function of a smart contract, she has to pay the total gas cost associated with the operations executed by the contract. The user includes a prepayment (deposit) of gas with her transaction. If the paid deposit is insufficient, the transaction will be rejected and reverted [9, 47].

Attacks on Ethereum Contracts. There are a variety of security vulnerabilities and possible attacks on Ethereum contracts [2]. A well-known family of these attacks are called gas limit attacks. Every Ethereum block is limited to handling at most a specific amount of computation, known as the gas limit. If a function call consumes more gas than the gas limit, it will fail and the consequences are often mishandled by smart contract programs [20]. Hence, a common best-practice is to avoid writing codes that can have an unbounded

runtime. In practice, given the cost of gas, one should aim to develop smart contracts that execute as little computation as possible. Many real-world smart contracts do not even have loops [20].

Solidity. Solidity is a programming language for writing Ethereum smart contracts [48]. It was developed by the Ethereum team in 2014 and is currently the most widely-used smart contract programming language. Solidity aims to provide the programmer with all the usual functionality of a general-purpose language like C++.

Vyper. Vyper is the newest language of the Ethereum foundation [26]. It is a python-like scripting language whose goal is to provide a simple way of writing secure real-world smart contracts, by disallowing vulnerable functionality, and hence losing Turing-completeness, in exchange for more security [26]. One of the decisions by the designers of Vyper was to disallow the rarely-used functionality of infinite loops and recursion, in order to avoid gas limit attacks. Hence, all Vyper smart contracts are simple programs [26].

Types of Termination. A function in a smart contract can terminate in a number of ways. On Ethereum, the possibilities are [9, 47]:

- (i) *Return:* As in classical programs, the function can terminate by returning control to its parent function.
- (ii) *Revert:* The function can terminate by canceling the entirety of the current transaction, e.g. when an error occurs and the whole transaction must be rolled back. In this case, all the changes made by the current transaction, including those made by other functions, possibly even by other contracts, will be reverted. There are essentially two types of reversion: the programmer can choose to either refund the remaining gas after reversion or to burn it. These correspond to the `require` and `assert` keywords in Solidity.
- (iii) *Self-destruct:* Finally, a function can terminate by destructing the current contract, making it unusable in the future, i.e. the functions of the contract will no longer be callable by anyone. Self-destruction is usually used when a contract reaches its expiration and is no longer useful, or when serious errors or security problems happen and it is necessary to stop any further interaction with the contract. In this case, the balance of the contract will be transferred to a predefined recipient.

Modeling Vyper and Solidity Contracts in ES. In our language, ES, we use `return` to model termination by returning and `exit1`, `exit2` and `exit3` to model the other types of termination. A Solidity or Vyper `while(ϕ)` loop can simply be modeled by an ES loop whose body begins with `if(! ϕ) then break; endif`. Other types of loops, such as `for` can be modeled similarly. Hence, to prove that Solidity and Vyper smart contracts have CFGs (or GCFCGs) of bounded treewidth, it suffices to show the same fact for ES.

Remark. In this work, we are considering structured, i.e. goto-free, programs. Therefore, we do not consider Solidity programs that include the so-called Solidity assembly code. It is well-known that one can write assembly codes that have arbitrarily large treewidth [31].

2.3 Motivating Examples

Tree decompositions and treewidth are formally defined in the next section. In this section, we provide some motivating examples to illustrate the importance of treewidth boundedness.

Formal Analysis. Exploiting graph structures for obtaining faster analysis algorithms is a well-studied field [3]. Bounded treewidth is one of the most widely-used structures and leads to efficient algorithms for many formal analysis problems [28]. The usual approach to formal program analysis is to write the desired property of the program in a specification language or logic. For example, in case of smart contracts, a desired property might be that one party cannot cause a self-destruction of the contract if another party opposes it. As another example, to avoid the DAO attack, we can specify the property of avoiding the reentrancy vulnerability over all runs of the contract. Two of the most commonly-used formal languages for specifying desired properties are the μ -calculus and the Monadic Second Order Logic (MSO). There is no known polynomial-time algorithm for the problem of μ -calculus model checking, i.e. checking whether a given program/contract satisfies a property specified in μ -calculus, but the problem can be solved in linear time if the CFG has constant treewidth [39]. Similarly, model checking MSO properties is NP-hard in general, but can be done in linear time if the underlying graph has constant treewidth [35].

Compiler Optimization. A classical and well-studied problem in compiler optimization is that of register allocation [33], i.e. assigning program variables to a limited number of registers in an optimal manner. Register allocation is one of the most important stages for optimizing several typical goals, such as energy efficiency, code size and execution speed [33]. This problem is usually reduced to graph coloring, which is NP-hard even for 3 colors (equivalent to 3 registers) [45]. However, if the CFG has constant treewidth, then register allocation can be solved in polynomial time [32, 45]. In case of smart contracts, such optimizations at compile time can significantly reduce the gas costs and the overall energy that is used by the network to run a contract.

Quantitative Analysis. In contrast with classical verification, which classifies a program as either correct or incorrect, quantitative analysis assigns a value to every run of the program that quantifies the cost/revenue generated by that run [18]. In case of smart contracts, this value can naturally model financial gains or losses of a party in the contract, or the amount of gas/energy used by the contract. Hence, quantitative analysis of smart contracts is a natural and important problem [16]. In [18], it was shown that treewidth can help significantly in speeding up several quantitative analyses. Hence, treewidth boundedness leads to much faster algorithms for analyzing the economic effects of a smart contract.

2.4 Tree Decompositions and Treewidth

In this section, we succinctly review the notions of treewidth and tree decomposition. For a more in-depth treatment see [22].

Tree Decompositions. Consider a graph $G = (V, E)$. A tree decomposition of G is a pair $(T, \{X_t | t \in T\})$ where T is a tree and every node t of T is labeled by a subset $X_t \subseteq V$ of vertices of G , such that the following conditions are satisfied:

- Every vertex $v \in V$ must appear in at least one X_t , i.e. $\cup_{t \in T} X_t = V$;
- For every edge $\{u, v\} \in E$, there must exist an X_t containing both u and v , i.e. $\forall e \in E \exists t \in T e \subseteq X_t$;
- For each vertex $v \in V$, the set $T^v = \{t \in T | t \in X_t\}$ must be a connected subtree of T . Note that T^v is the set of all

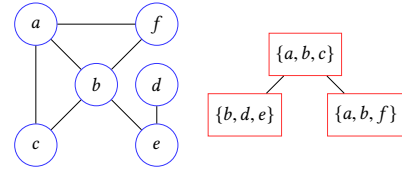


Figure 3: A graph G (left) and one of its optimal tree decompositions $(T, \{X_t\})$ (right).

nodes of T that contain v in their corresponding X_t . Hence, this condition means that every vertex should appear in a connected subtree of T .

We reserve the word “vertex” for vertices of G and use the word “node” to refer to vertices of the tree T . Also, we call each X_t a “bag”.

Treewidth. The width of a tree decomposition $(T, \{X_t\})$ is defined as the size of the largest bag minus 1, i.e. $w(T, \{X_t\}) := \max_{t \in T} |X_t| - 1$. The treewidth $tw(G)$ of a graph G is defined as the smallest width among all tree decompositions of G .

Example. Figure 3 shows a graph G and one of its tree decompositions. This decomposition has a width of 2. It is easy to verify that G cannot have a tree decomposition of width 1. Hence, this tree decomposition is optimal and the treewidth of G is 2.

Dynamic Programming Algorithms. The significance of treewidth and tree decompositions in algorithm design stems from the fact that many hard graph problems can be solved in polynomial (often linear) time by performing a bottom-up dynamic programming on the tree decomposition, in essentially the same manner that is employed for solving problem on trees [6, 7, 22, 23]. A main concept in these algorithms is that one can associate a subgraph of G to every node of T . To do so, we fix an arbitrary node r as the root of T . Then, for each node $t \in T$, we let its corresponding subgraph G_t consist of all the vertices that appear in the bags of the subtree of T rooted at t , i.e. either in X_t or in the bags of its descendant nodes. Similarly, the edges of G_t are those edges of G that have both their endpoints appearing together in some bag in the subtree rooted at t . Hence, $G_r = G$ and if a node t has children t_1, t_2, \dots, t_k , then for all t_i , we have $G_{t_i} \subseteq G_t$. The basic idea is then to compute the answer(s) to the problem at G_t by means of divide-and-conquer based on the answer(s) at G_{t_i} 's.

We now provide a different but equivalent formulation of the notion of treewidth as in [45]. We will make use of both formulations in our proofs in Sections 3 and 4.

Listings. Given a graph $G = (V, E)$, a listing L is simply a permutation of the vertices of G , i.e. a sequence of elements of V in which every $v \in V$ appears exactly once.

Separators. Given a graph $G = (V, E)$, a listing L and a vertex $v \in V$, let $l(v)$ be the set of all vertices that appear before v in the listing L and $r(v)$ be the set of all vertices that appear after v . Then the separator of v is the set of all vertices in $l(v)$ that can be reached from v using a path whose internal vertices are all in $r(v)$. We use the notation S_v^L , or simply S_v when L is clear from the context, to denote the separator of v .

Complexity of Listings and Graphs. The complexity of a listing L is defined as the size of its largest separator, i.e. $c(L) := \max_{v \in V} |S_v^L|$.

The complexity $c(G)$ of a graph G is defined as the minimum complexity among all its listings.

Example. Consider the graph in Figure 3 together with the listing $L = \langle a, b, c, d, e, f \rangle$. We have the following separators: $S_a = \emptyset, S_b = \{a\}, S_c = \{a, b\}, S_d = \{b\}, S_e = \{b, d\}, S_f = \{a, b\}$. Hence, L has complexity 2. One can also verify that G has no listing of a lower complexity, hence $c(G) = 2$.

We now review some previous lemmas and results.

LEMMA 2.1. *For every graph G , we have $c(G) = tw(G)$, i.e. the complexity of a graph is the same as its treewidth [24]. Moreover, there is an algorithm to obtain a tree decomposition of width k from a listing of complexity k in linear time [45].*

LEMMA 2.2. [Contraction Lemma] *Consider a graph $G = (V, E)$ of treewidth k and an edge $\{u, v\} \in E$. Let G' be the graph obtained by contracting $\{u, v\}$ in G . Then $tw(G') \leq tw(G)$. Moreover, there is a linear-time algorithm that given a listing of G with complexity k , produces a listing of G' with complexity at most k [45].*

Treewidth of Control Flow Graphs. In [45], it was shown that CFGs of goto-free Algol, Pascal and C programs have a treewidth of at most 6. In [31] it was shown that there are Java programs with arbitrarily large treewidth, but real-world Java codes typically have a treewidth of 2 or 3. In [8] a similar result was obtained for Ada programs.

3 INTRAPROCEDURAL TREEWIDTH OF SMART CONTRACTS

In this section, we consider the CFGs of ES programs and show that they always have bounded treewidth. As argued in Section 2.2, Solidity and Vyper programs can be modeled in ES and hence a treewidth boundedness result for ES naturally extends to contracts written in these languages. We closely follow the construction in [45], with modifications for hexagonal nodes, and provide a natural way of obtaining a listing from an ES program. Finally, we compute an upperbound for the complexity of this listing, hence bounding the treewidth using Lemma 2.1.

Notation. Consider the CFG G_f of an ES function f and let v be a vertex in G_f . We use v^- (resp. v^+) to denote the predecessor (resp. successor) of v in G_f . If there are more than one successor or predecessor, we will take the one that is not inside the block of v . So, a return^+ is the semicolon following a return and a loop^- is the vertex before that loop (and not the last vertex inside the block of the loop). Moreover, if v corresponds to an atomic boolean variable, then we use T_v (resp. F_v) to denote the vertex of the CFG that will be visited after v if its value is True (resp. False). See [11] for a list of potential neighbors vertices based on their type.

Canonical Listing of an ES Function. Given an ES function f and its CFG G_f , the canonical listing $CL(f)$ is a listing that visits the vertices of G_f in the following recursive manner:

- If the function is of the form $\text{function } f \text{ A endfunction}$, we first visit endfunction, followed by function, the hexagonal nodes $1, \dots, k$, and finally a recursive visiting of A .
- If A is of the form $B ; C$, we first visit the semicolon, followed by a recursive visit of B and then a recursive visit of C .

- If A is of the form $B C$, where B is a statement that has an “end” vertex, i.e. if B is either an if-then(-else)-endif statement or a loop-endloop statement, we first visit B and then C , both recursively².
- If A is of the form $\text{if } B \text{ then } C \text{ else } D \text{ endif}$, we visit it in this order: endif, if, then, else, B, C, D . Where the visits to B, C and D are recursive. Similarly, if A is of the form $\text{if } B \text{ then } C \text{ endif}$, the visiting order would be endif, if, then, B, C .
- If A is of the form $\text{loop } B \text{ endloop}$, we first visit endloop, followed by loop and a recursive visit of B .
- If A is a boolean expression of the form $B \text{ or } C$, we first visit or and then visit B and C recursively. We do the same for B and C .
- If A is an atomic boolean variable or statement, we just visit it, i.e. add it to the listing.

Intuitively, we visit the parts of the program in a top-down fashion.

Separators. We now find the separators of every vertex in the CFG G_f with respect to the canonical listing $CL(f)$. The vertex endfunction appears in the beginning of the listing, so $S_{\text{endfunction}} = \emptyset$. Given that endfunction is the only vertex appearing before function, we have $S_{\text{function}} \subseteq \{\text{endfunction}\}$. For a word (vertex) v , we use \underline{v} (resp. \bar{v}) to denote the word preceding (resp. succeeding) v in the ES program³. Similarly, if X is a set of consecutive words of the program, we use \underline{X} (resp. \bar{X}) to denote the word exactly before (resp. after) X . Now consider a vertex v ,

- If v is an if, then the separator of v can include \underline{v} and endif because they are connected to v in the CFG and appear before v in the listing. However, if v is inside a loop, the separator can also contain loop and endloop due to the possibility of existence of continue and break statements in the blocks of v . Similarly, exit and return statements make it possible for the separator to contain the hexagonal nodes $1, 2, \dots, k$ and the vertex endfunction. Hence we have $|S_v| \leq k + 5$. Recall that k is the number of different exit types.
- The words then, else and endif have almost the same situation. If v is a vertex corresponding to the word then, we have $S_v \subseteq \{\text{if, endif, loop, endloop, endfunction, } 1, \dots, k\}$. Also, if v is a vertex corresponding to the word else, then $S_v \subseteq \{\text{if, then, endif, loop, endloop, endfunction, } 1, \dots, k\}$. For the case of endif, we have $S_v \subseteq \{\text{if, loop, endloop, endfunction, } 1, \dots, k\}$.
- If v is a loop, then $\underline{v} = v^-$, hence \underline{v} is a neighbor of v in the CFG and appears before v in the listing. Therefore \underline{v} is in the separator. So $S_v \subseteq \{\underline{v}, \text{endloop, endfunction, } 1, \dots, k\}$.
- If v is an endloop and u is the corresponding loop, then $\underline{u} \in S_u$ and hence $\underline{u} \in S_v$. So we have $S_v \subseteq \{\underline{u}, \text{loop}', \text{endloop}', \text{endfunction, } 1, \dots, k\}$. Here, $\text{loop}'\text{-endloop}'$ is the higher level loop containing u and v (if such a loop exists).

²If there are several ways of writing A as $B ; C$ or $B C$, we choose the shortest B .

³Note that these are not necessarily the same as v^- and v^+ which are defined using the CFG, not the order of the words in the ES program

- If v corresponds to either of atomic, break, continue, call or return, it is easy to see that $v^+ = \bar{v}$ and $v^- = \underline{v}$ and they are both visited before v . Hence, $S_v = \{v^+, v^-\}$.
- If v is a break^+ , i.e. the semicolon after a break vertex v^- , then v^{--} appears before v in the listing and the path $v \rightarrow v^- \rightarrow v^{--}$ exists in the CFG. So if v^{--} exists, then $v^{--} \in S_v$. Hence, $S_v \subseteq \{v^{--}, \text{endloop}\}$, because in the CFG we jump straight to endloop after v . Similarly, we have $S_v \subseteq \{v^{--}, \text{loop}\}$ (resp. $S_v \subseteq \{v^{--}, \text{endfunction}\}$) if v is a continue^+ (resp. return^+). However, if v is atomic^+ or call^+ , then the execution of the function continues as usual after v , so $S_v \subseteq \{v^{--}, \text{loop}, \text{endloop}, \text{endfunction}, 1, \dots, k\}$.
- If v is an and vertex in A and B , then it is easy to check that $S_v \subseteq \{\text{if}, \text{then}, \text{else}, \text{endif}, \underline{A}, \underline{B}\}$. The same holds if v is an or vertex. Similarly, if v is an atomic boolean variable, then $S_v \subseteq \{T_v, F_v, \underline{v}, \bar{v}\}$.
- Finally, if i is a hexagonal exit node, then $S_i \subseteq \{\text{function}, \text{endfunction}, 1, 2, \dots, k\}$.

Hence, we have the following lemma and corollary:

LEMMA 3.1. *For every ES function f with k exit types, the canonical listing $CL(f)$ of the CFG G_f has a complexity of at most $k + 6$.*

COROLLARY 3.1. *The CFG of any ES function f with k exit types has a treewidth of at most $k + 6$.*

REMARK 1 (SHARPNESS). *The bound obtained in Corollary 3.1 is sharp, i.e. for every k , one can obtain an ES programs with k exit types and a treewidth of exactly $k + 6$, by ensuring that for any vertex type, there is a vertex v in the CFG whose separator S_v includes all the possible cases enumerated above.*

We are now ready for the main theorem of this section.

THEOREM 3.1. *The CFG of every Solidity or Vyper smart contract has a treewidth of at most 9.*

PROOF. We use corollary 3.1. As shown in Section 2.2, Solidity and Vyper programs can be modeled in ES with 3 exit types, i.e. revert with/without gas refund and self-destruct. \square

Remark. Note that our approach is constructive and we provided a linear-time algorithm for obtaining a 9-complex listing by one pass over the code of the smart contract. This, together with the algorithm of Lemma 2.1, ensure that one can obtain a tree decomposition of width 9 from the contract code in linear time.

4 INTERPROCEDURAL TREewidth OF SMART CONTRACTS

In this section, we consider the treewidth of GCFGs of smart contracts. If we are given a constant-width tree decomposition of the GCFG of a contract, then we can naturally apply dynamic programming algorithms for solving interprocedural (global) problems. Intuitively, given that CFGs of smart contracts have constant treewidth and GCFGs are obtained by piecing copies of CFGs together in a structured manner, it should come as no surprise that GCFGs have constant treewidth, too. We formally prove this in Theorem 4.1.

Given this treewidth boundedness result, the only remaining challenge for applying dynamic programming algorithms is the

size of the GCFG and the resulting tree decomposition. For recursive programs, the GCFG is infinite. Fortunately, as mentioned in Section 2.2, all Vyper smart contracts are simple programs. On the other hand, while recursion is possible in Solidity, our experimental results (Section 5) show that real-world Solidity contracts are simple programs as well. Unfortunately, even for simple programs, the GCFG, and hence its tree decompositions, can have exponential size with respect to the length of the program. We overcome this difficulty by designing a succinct representation of tree decompositions that (i) can be directly computed from the program code in linear time, and (ii) is compatible with bottom-up dynamic programming algorithms, allowing them to run in polynomial time (with respect to the length of the program, rather than the size of the GCFG) by eliminating unnecessary repeated computations. We now prove interprocedural treewidth boundedness.

THEOREM 4.1. *The GCFG of every Solidity or Vyper smart contract has a treewidth of at most 10.*

PROOF. Let C be a Solidity or Vyper smart contract with functions f_1, f_2, \dots, f_n . Also, let G be the GCFG of C and G_i be the CFG of the function f_i . Theorem 3.1 guarantees that every G_i has a treewidth of at most 9. Let $\tau_i = (T_i, \{X_t | t \in T_i\})$ be a tree decomposition of G_i with width at most 9 and τ_i^+ be a tree decomposition of G_i , obtained by adding the endfunction vertex to every bag, i.e. $\tau_i^+ = (T_i, \{Y_t | t \in T_i\})$ where $Y_t = X_t \cup \{\text{endfunction}\}$. We show how to create a tree decomposition $\tau = (T, \{X_t | t \in T\})$ of width at most 10 of G using the τ_i 's and τ_i^+ 's. The process mimics the procedure for creating G using the G_i 's. We start with an unexpanded graph G^0 and a tree decomposition τ^0 of G^0 . In each step, we expand the call vertices in G^k to obtain G^{k+1} . We also create a new tree decomposition τ^{k+1} . The invariant satisfied by the algorithm is that for every k , the obtained τ^k is always a tree decomposition of G^k of width at most 10.

- (1) Let $G^0 = \bigcup_{i=1}^n G_i$ and $\tau^0 = \bigcup_{i=1}^n \tau_i$. Also, set an arbitrary node in every connected component of τ^0 as the root.
- (2) While there is an unexpanded call vertex in G^k :
 - Let $G^{k+1} = G^k$.
 - For every unexpanded call vertex of G^k of the form call f_i ; appearing in the function f_j :
 - Expand the call in G^{k+1} .
 - Let t be a node in τ^k such that the bag X_t contains both the call and the semicolon following it. Note that such a node must exist because the call and the semicolon are neighbors in G^k and τ^k is a tree decomposition of G^k . Also, let s be a node in τ_i^+ whose bag X_s contains the vertex function.
 - Create a new node t' in τ^{k+1} , connect it to t as a child, and let $X_{t'} = \{\text{call}, ;, \text{function}, \text{endfunction}\}$. This bag contains two elements from X_t and two from X_s . We call t' an intermediary node.
 - Add a copy of τ_i^+ (rooted at s) to τ^{k+1} , i.e. let $\tau^{k+1} = \tau^k \sqcup \tau_i^+$, and connect t' to the node s in this copy such that t' is the parent and s is the child.

It is easy to check that the procedure above satisfies the invariant. Clearly, τ^0 is a tree decomposition of G^0 with width at most 9. In the process of expanding a function call in G^{k+1} , all the new vertices

and edges between them are covered in τ^{k+1} by the new copy of τ_i^+ , which has a width of at most 10. The edges from call to function and from endfunction to the semicolon appear in $X_{t'}$.

Finally, in case of simple programs, the process ends at some point. Hence there is a G^k such that $G^k = G$. So τ^k is the desired τ . In case of recursive programs, we have $G = \bigcup_{i=0}^{\infty} G^i$ and can hence define $\tau := \bigcup_{i=0}^{\infty} \tau^i$. This completes the proof. \square

Succinct Representation of Interprocedural Tree Decompositions. We now consider simple programs only. Note that the tree decomposition τ created in the process above has a lot of redundancy. Basically, τ is obtained by piecing together the τ_i 's, which are tree decompositions of the CFGs of the functions f_i , with many copies of the τ_i^+ 's and intermediary nodes (the t' nodes in the proof above). We can eliminate this redundant copying without affecting the results of dynamic programming algorithms. Formally, let t be a node in τ_i^+ (or an intermediary node), then it is easy to verify that, by construction, every copy of t in τ has the same associated subgraph $G_t \subseteq G$ (up to the natural isomorphism) and the same bag X_t . Therefore, in a dynamic programming scheme, we can only compute the answers in one of the copies of G_t , and reuse them for all other copies. Equivalently, we can represent the rooted tree decomposition $\tau = (T, \{X_t | t \in T\})$ in a succinct manner $\tau^* = (T^*, \{X_{t^*} | t^* \in T^*\})$ by merging all copies of the same τ_i^+ (or intermediary node) into one. This of course means that the same node can now have several parents and T^* is a DAG, i.e. directed acyclic graph, instead of a rooted tree. However, the dynamic programming algorithms can be applied to τ^* in the same manner as in τ , i.e. in bottom-up order⁴. This automatically avoids the redundant and repetitive computations at every copy of τ_i^+ .

THEOREM 4.2. *The GCFG of every simple Vyper or Solidity smart contract has a tree decomposition of width at most 10 that is compatible with bottom-up dynamic programming algorithms and can be succinctly represented in linear size, with respect to the length of the contract. Moreover, this succinct representation can be obtained from the contract code in linear time.*

PROOF. We take the tree decomposition τ and its succinct representation τ^* as described above. We have already shown the width and compatibility with bottom-up dynamic programming. We just need to prove that τ^* has linear size and can be obtained in linear time. Each of the one-function tree decompositions τ_i and τ_i^+ appear exactly once in τ^* and there are at most as many intermediary nodes as the number of call operations in the code. Hence, τ^* has linear size. Also, the same process that was used for obtaining τ in the proof of Theorem 4.1 can be applied to obtain τ^* , except that every call site should be expanded only once. Hence, τ^* can be computed in linear time. \square

5 EXPERIMENTAL RESULTS

Our Tool. We implemented a tool in Python/C++ that gets a Solidity smart contract as input and outputs its canonical listing, tree decompositions of its intraprocedural CFGs (with width at most 9)

⁴Note that the simplicity assumption is indeed necessary. If the program is recursive, then T^* contains a cycle and is no longer a DAG. Hence, there is no bottom-up ordering of the nodes and dynamic programming algorithms cannot be applied.

and the succinctly-represented tree decomposition τ^* of its interprocedural GCFG (with width at most 10). Our tool works in linear time and is very efficient in practice. We use the ConsenSys Solidity parser [21] to obtain the CFGs.

Benchmarks. We used the contracts listed in the Etherscan database of verified Solidity source codes [44] as our benchmarks. Ethereum contracts are saved on the Blockchain in Bytecode format, but many users like to see the actual Solidity code of the contract. Hence, Etherscan allows programmers to publish the original Solidity code, then compiles it and verifies that the resulting bytecode is the same as the one published on the Blockchain. Hence, all of our benchmarks are real-world smart contracts that are currently deployed on the Ethereum Blockchain.

Number of Benchmarks. At the time of writing, there are just below 40,000 smart contracts listed in the Etherscan database. Of these, we ignored contracts that include assembly code and hence can have arbitrarily large treewidth, and the ones that produced compilation errors. This left us with 36,764 benchmarks.

Runtime and Machine. We used an Intel Core i5-7200U 2.5GHz Processor running Ubuntu 18.04. We ran our approach on all 36,764 benchmarks. In all cases, our runtime was less than 0.1 seconds.

Intraprocedural Results. We found that for CFGs of real-world smart contracts, the treewidth bound 9 is never met. The highest width among the obtained tree decompositions was 6 and the average was 3.35. Figure 4 shows the number of contracts with each width. Note that the y -axis is in log scale and a vast majority of obtained tree decompositions have a width of 3 or 4.

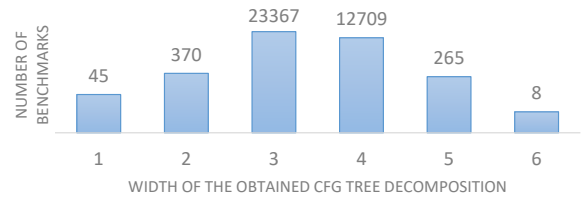


Figure 4: Widths of tree decompositions of CFGs

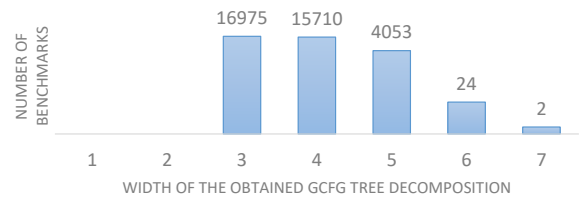


Figure 5: Widths of tree decompositions of GCFGs.

Interprocedural Results. We found that all our benchmarks are simple programs and recursion is not used in real-world Solidity smart contracts. Similar to the previous case, the bound 10 was never met in practice. The highest width of a GCFG tree decomposition in our benchmarks was 7 and the average width was 3.65. Figure 5 shows the number of contracts with each interprocedural width. Note that the y -axis is in log scale and a vast majority of

contracts have an interprocedural tree decomposition of width 3 or 4. However, note that in our construction of τ (Theorem 4.1), the intermediary nodes have a bag of size 4, so the tree decompositions have a width of at least 3. See [11] for more detailed experimental results.

6 CONCLUSION

In this paper, we showed that Ethereum smart contracts written in Solidity and Vyper have small treewidth. We obtained a theoretical bound of 9 for the intraprocedural treewidth and 10 for the interprocedural case. We also reported on a tool we implemented for computing treewidth of Solidity contracts and provided experimental results that showed the treewidth of real-world contracts is often much smaller. We argued that the treewidth boundedness result can be exploited to obtain much faster algorithms for program analysis, model checking, compiler optimization and quantitative analysis of contracts. A natural next step would be to develop analysis and optimization tools for smart contracts using the currently-known faster algorithms that exploit treewidth.

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